

# VZOROVKA A

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①  
Kene

1. a)

$$f_1: y = \frac{1}{\log_2(x+4) - 3}$$

$$x+4 > 0$$

$$x > -4$$

$$\log_2(x+4) - 3 \neq 0$$

$$\log_2(x+4) \neq 3$$

$$\log_2(x+4) \neq \log_2 8$$

$$x+4 \neq 8$$

$$x \neq 4$$

$$D_{f_1} = (-4; 4) \cup (4; +\infty)$$

b)

$$f_2: y = \log(x^2 - 4)$$

$$x^2 - 4 > 0$$

$$(x-2) \cdot (x+2) > 0$$

$$\downarrow \quad \downarrow$$
$$x_1 \neq 2 \quad x_2 \neq -2$$

x-2	-	-2	-	2	+
x+2	-		+		+
	⊕		⊖		⊕

$$D_{f_2} = (-\infty; -2) \cup (2; +\infty)$$

2.

$$4^x - 3^x \cdot 3^{-\frac{1}{2}} = 3^x \cdot 3^{\frac{1}{2}} - 2^{2x} \cdot 2^{-1}$$

$$2^{2x} - 3^x \cdot 3^{-\frac{1}{2}} = 3^x \cdot 3^{\frac{1}{2}} - 2^{2x} \cdot 2^{-1}$$

$$2^{2x} + 2^{2x} \cdot 2^{-1} = 3^x \cdot 3^{\frac{1}{2}} + 3^x \cdot 3^{-\frac{1}{2}}$$

$$2^{2x} \cdot \frac{3}{2} = 3^x \cdot \frac{4\sqrt{3}}{3}$$

$$\frac{2^{2x}}{3^x} = \frac{4\sqrt{3}}{\frac{3}{2}}$$

$$\frac{2^{2x}}{3^x} = \frac{8\sqrt{3}}{9}$$

$$\frac{2^{2x}}{2^x} = \frac{2^3 \cdot 3^{\frac{1}{2}}}{3^2}$$

$$\frac{2^{2x}}{3^x} = \frac{2^3}{3^{\frac{3}{2}}}$$

$$\frac{2^{2x}}{3^x} = \left(\frac{2^3}{3}\right)^2$$

$$\left(\frac{2^2}{3}\right)^x = \left(\frac{2^3}{3}\right)^2$$

$$\left(\frac{4}{3}\right)^x = \left(\frac{8}{3}\right)^2$$

$$x = 2 \log_{\frac{4}{3}} \left(\frac{8}{3}\right)$$

$$4^x - 3^x \cdot 3^{-\frac{1}{2}} = 3^x \cdot 3^{\frac{1}{2}} - 2^{2x} \cdot 2^{-1}$$

$$2^{2x} - 3^x \cdot \frac{1}{\sqrt{3}} = 3^x \cdot \sqrt{3} - 2^{2x} \cdot \frac{1}{2}$$

$$1 - \frac{3^x}{\sqrt{3} \cdot 2^{2x}} = \frac{3^x \sqrt{3}}{2^{2x}} - \frac{1}{2}$$

$$1 - \left(\frac{3}{2^2}\right)^x \cdot \frac{1}{\sqrt{3}} = \left(\frac{3}{2^2}\right)^x \sqrt{3} - \frac{1}{2}$$

$$1,5 = \left(\frac{3}{2^2}\right)^x \cdot \left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$$

$$\frac{3}{2} = \left(\frac{3}{2^2}\right)^x \cdot \left(\frac{4}{\sqrt{3}}\right)$$

$$\frac{3\sqrt{3}}{2^3} = \left(\frac{3}{2^2}\right)^x$$

$$\frac{3^{\frac{3}{2}}}{2^3} = \left(\frac{3}{2^2}\right)^x$$

$$\frac{3^{\frac{3}{2}}}{2^3} = \frac{3^x}{2^{2x}}$$

$$\frac{\sqrt{3^3}}{8} = \frac{3^x}{4^x}$$

$$\frac{3\sqrt{3}}{8} = \frac{3^x}{4^x}$$

$$\left(\frac{3}{4}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^x$$

$$x = \frac{3}{2}$$

$$X = \left\{\frac{3}{2}\right\}$$

3.

$$\frac{3}{2} \log \frac{x^2}{10} + \log \frac{100}{x^3} - \log \frac{\sqrt{10}}{x} = -2$$

$$\begin{array}{l} x > 0 \\ D_f = (0; +\infty) \end{array}$$

$$\frac{3 \log \frac{x^2}{10}}{2} + \log 100 - \log x^3 - (\log \sqrt{10} - \log x) = -2$$

$$\frac{3(\log x^2 - \log 10)}{2} + \log 10^2 - \log x^3 - (\log 10^{\frac{1}{2}} - \log x) = -2$$

$$\frac{3(\log x^2 - 1)}{2} + 2 - \log x^3 - \left(\frac{1}{2} - \log x\right) = -2$$

$$\frac{3 \log x^2 - 3}{2} + 2 - \log x^3 - \frac{1}{2} + \log x = -2$$

$$\frac{6 \log x - 3}{2} + 2 + \log \frac{x}{x^3} - \frac{1}{2} = -2$$

$$\frac{6 \log x - 3}{2} + 2 + \log \frac{1}{x^2} - \frac{1}{2} = -2$$

$$\frac{6 \log x - 3}{2} + 2 + \log x^{-2} - \frac{1}{2} = -2$$

$$\frac{6 \log x - 3}{2} + 2 - 2 \log x - \frac{1}{2} = -2$$

$$\frac{6 \log x - 3}{2} - 2 \log x + \frac{3}{2} = -2 \quad /:2$$

$$6 \log x - 3 - 4 \log x + 3 = -4$$

$$2 \log x = -4 \quad /:2$$

$$\log x = -2$$

$$x = 10^{-2}$$

$$x = \frac{1}{100} \in D_f$$

$$K = \left\{ \frac{1}{100} \right\}$$

4.

$$g: y = -|\log_{\frac{1}{2}}(x+2)|$$

$$x > -2$$

Px

$$0 = -|\log_{\frac{1}{2}}(x+2)|$$

$$-|\log_{\frac{1}{2}}(x+2)| = 0$$

$$-|-\log_2(x+2)| = 0$$

$$|-\log_2(x+2)| = 0$$

$$-\log_2(x+2) = 0$$

$$\log_2(x+2) = 0$$

$$x+2 = 1$$

$$x = -1$$

$$P_x[-1; 0]$$

Py

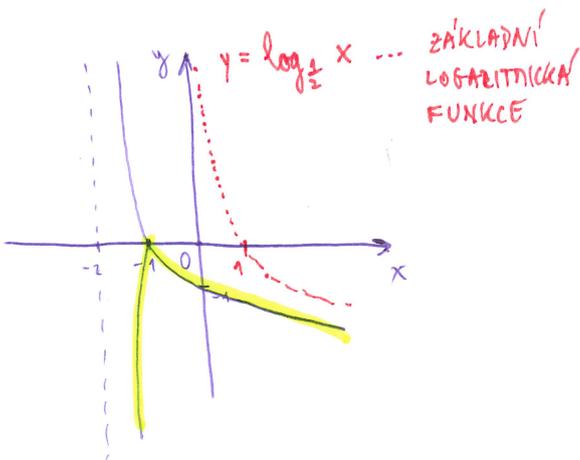
$$y = -|\log_{\frac{1}{2}}(0+2)|$$

$$y = -|\log_{\frac{1}{2}} 2|$$

$$y = -|-1|$$

$$y = -1$$

$$P_y[0; -1]$$



$$D_f = (-2; \infty)$$

$$H_f = (-\infty; 0)$$

$$P_x[-1; 0]$$

$$P_y[0; -1]$$

asymptota  $\{-2\} \dots x = -2$

omeznenost: shora ano } nemí omezena  
zdola ve

rostoucí  $(-2; -1)$

klesající  $(-1; \infty)$

min: nemá

max:  $[-1; 0]$

ani sudá

ani lichá

nemí prostá

5.  $f: y = \left(\frac{2m-1}{3m+2}\right)^x$

1. podmínka:

$$\frac{2m-1}{3m+2} > 0$$

$$2m-1 = 0$$

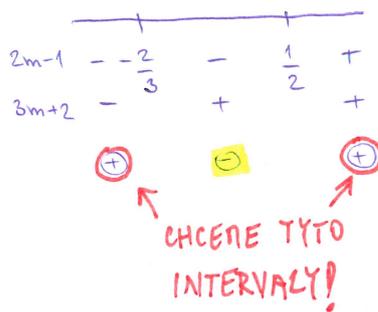
$$2m = 1$$

$$m = \frac{1}{2}$$

$$3m+2 = 0$$

$$3m = -2$$

$$m = -\frac{2}{3}$$



2. podmínka:

$$\frac{2m-1}{3m+2} < 1$$

$$\frac{2m-1}{3m+2} - 1 < 0$$

$$\frac{2m-1-1 \cdot (3m+2)}{3m+2} < 0$$

$$\frac{2m-1-3m-2}{3m+2} < 0$$

$$\frac{-m-3}{3m+2} < 0$$

$$-m-3 \neq 0$$

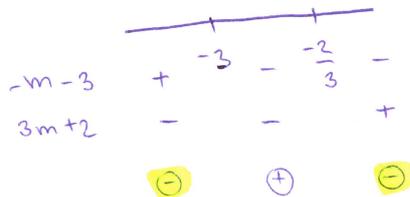
$$-m \neq 3$$

$$m \neq -3$$

$$3m+2 \neq 0$$

$$3m \neq -2$$

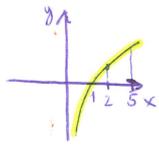
$$m \neq -\frac{2}{3}$$



~~$$m \in \left(-\frac{2}{3}; \frac{1}{2}\right)$$~~

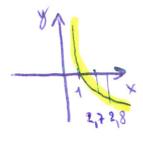
$$m \in (-\infty; -3) \cup \left(\frac{1}{2}; \infty\right)$$

6.  $\log_a 2 < \log_a 5$



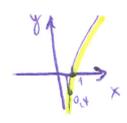
$a > 1$

$\log_a 2,7 > \log_a 2,8$



$a \in (0, 1)$

$\log_a 0,4 < \log_a 1$



$a > 1$

7.

$$\frac{\sqrt{y^{-\frac{1}{3}} \sqrt{y^{\frac{4}{3}}}} \cdot \sqrt{a \cdot \sqrt{y}}}{\sqrt[3]{a \cdot y}} =$$

$$= \frac{y^{\frac{1}{2} \cdot (-\frac{1}{3})} \cdot y^{\frac{1}{3}} \cdot a^{\frac{1}{2}} \cdot y^{\frac{1}{3} \cdot \frac{1}{2}}}{a^{\frac{1}{3}} \cdot y^{\frac{1}{3}}} =$$

$$= \frac{y^{-\frac{1}{6}} \cdot y^{\frac{1}{3}} \cdot a^{\frac{1}{2}} \cdot y^{\frac{1}{6}}}{a^{\frac{1}{3}} \cdot y^{\frac{1}{3}}} =$$

$$= \frac{a^{\frac{1}{2}}}{a^{\frac{1}{3}}} = a^{\frac{1}{2}} \cdot a^{-\frac{1}{3}} = \underline{\underline{a^{\frac{1}{6}}}}$$

$a > 0$
$y > 0$